Practical Witness-Key-Agreement for Blockchain-based Dark Pools Financial Trading

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Abstract. We introduce a new cryptographic scheme, Witness Key Agreement (WKA), that allows a party to securely agree on a secret key with a counter party holding publicly committed information only if the counter party also owns a secret witness in a desired (arithmetic) relation with the committed information.
Our motivating applications are over-the-counter (OTC) markets and dark pools, popular trading mechanisms. In such pools investors wish to communicate only to trading partners whose transaction conditions and asset holdings satisfy some constraints. The investor must establish a secure, authenticated channel with eligible traders where the latter committed information matches a desired relation. At the same time traders should be able to show eligibility while keeping their financial information secret.
We construct a WKA scheme for languages of statements proven in the designated-verifier Succinct Zero-Knowledge Non-Interactive Argument of Knowledge Proof System (zk-SNARK). We illustrate the practical feasibility of our construction with some arithmetic circuits of practical interest by using data from US Dollar denominated corporate securities traded on Bloomberg Tradebook.

Keywords: Blockchain-based dark pool; witness-key-agreement; zk-SNARK; quadratic arithmetic program; designated-verifier.

1 Introduction

Existing Blockchain-based Financial Systems Financial intermediation is traditionally based on trusted third party solutions, such as exchanges (e.g. NASDAQ or CME) or clearing mechanisms (e.g. EU's TARGET2-Securities and US's Depository Trust & Clearing Corporation).

New technologies have been recently proposed to replace these intermediaries with distributed protocols on blockchain. See for example ZeroCash [36], a cryptocurrency, or FuturesMEX [33], a crypto-based distributed futures exchange, or

⋆ This research was conducted during the author's visit to the University of Waterloo.

the dark pool exchange with three parties [11]. In those systems, the users commit financial information (e.g. accounts, bids and quotes) to a blockchain and use zero-knowledge proofs to show that their committed information satisfy a certain relation to preserve the integrity of the market and the solvency of the users. Noticeably, anonymity in those systems is as critical as confidentiality, e.g. the linkage of one's transactions can lead to strategic attacks against them [32].

New Dark Pools Requirements Private markets, i.e. dark pools, further reduce public information to protect large investors. The investor in a dark pool, who wants to sell at least v shares at price p, wants to disclose v and p only to traders who committed to have cash $c \geq pv$. Alternatively she might be willing to buy from somebody who has at least v' shares (an iceberg quote) or accept a price pegged within an interval, etc. For the very same reasons, the trader might not want to make his information fully public, but just to reassure the investor that he meets the constraints.

To make distributed dark pools possible, we propose *Witness Key Agreement* (WKA). In presence of a public blockchain holding parties' publicly committed information, WKA allows a party (the Verifier) to post a problem relation (e.g. a desired arithmetic or boolean combination of secret information) and securely agree on a secret key with another party (the Prover) holding a secret witness that *both* corresponds to the publicly committed information *and* satisfies the desired relation (i.e. the implicitly defined problem instance of the relation between the commits and the secret witness).

Witness Key Agreement Given n parties each having committed their private information ω and published the respective commitments ϕ anonymously on a public bulletin board, we consider the problem that a party wants to securely and anonymously agree on a secret key k with each counterparty based on their committed information ω. The initiating party wants to make sure that (and the key agreement is only successful if) the counterparty's committed information ω satisfies a public relation R (given by the initiating party), i.e. $R(\phi,\omega) = 1$, while each counterparty does not want to disclose their ω.

With our problem we push further the envelope of Non-Interactive Zero Knowledge (NIZK) [22]. In both cases, given an instance and an NP-relation R, a party (the Prover) can convince another party (the Verifier) that there exists a witness ω of the instance ϕ such that $R(\phi, \omega) = 1$, without leaking information about it. The successful outcome of NIZK is the binary verification result 1 while our desired outcome is a shared secret key.

Anonymity-Preserving Communication Model In our problem, we consider the anonymity of each party as critical as other WKA security properties. Therefore, our communication model assumes an anonymous network to hide the parties' identities (e.g., IP address) and all WKA communication must utilize the public bulletin board (e.g. a blockchain), i.e. to publish a message, a party sends it through the anonymous network to the public bulletin board which is readable by all parties.[5]

[5] WKA does not intend to hide whether the Prover/Verifier established communication as they are completely anonymous.

Table 1. Dark Pool Example Relations

In each relation we denote $[\![x]\!] = \mathsf{SHA256}(x; r_x)$ the public SHA256 commitment of the secret business variable x using randomness r_x. For a dark pool transaction we denote by c the cash capacity of a trader, c' the threshold given by the investor. For a bid we denote (p, v) as the bid price and the bid volume.

Sufficient Capacity (SC)	
Public $\phi = ([\![c]\!], c')$	Secret $\omega = (c, r_c)$
Conditions: $[\![c]\!] = \mathsf{SHA256}(c; r_c) \wedge c \geq c'$	
Price Range (PR)	
Public $\phi = ([\![p]\!], p'_+, p'_-)$	Secret $\omega = (p, r_p)$
Conditions: $[\![p]\!] = \mathsf{SHA256}(p; r_p) \wedge p'_- \leq p \leq p'_+$	
Matchable Bid (MB)	
Public $\phi = ([\![p]\!], [\![v]\!], p'_+, p'_-, c')$	Secret $\omega = (p, v, r_p, r_v)$
Conditions: $[\![p]\!] = \mathsf{SHA256}(p; r_p)$, $p'_- \leq p \leq p'_+ \wedge [\![v]\!] = \mathsf{SHA256}(v; r_v)$, $c' \geq pv$	

Practical WKA Construction We base our WKA construction on the concrete efficient construction of zk-SNARK from Non-Interactive Linear Proof (NILP) [24] for Quadratic Arithmetic Programs (QAP) [19] given by Groth [24] and we utilize Linear-Only Encryption (LE) [6] to compile such NILP to a WKA scheme. We provide the *first practical Witness Key Agreement under designated-verifier zk-SNARK proof for QAP*. In our WKA scheme construction a designated verifier can first broadcast a common reference string as a challenge for the relation R of interest. A prover can then publish a partial zk-SNARK proof as a response for the committed instance that satisfies R. Using the partial proof, the verifier can derive a shared secret key with the prover.

Non-goals The focus of our protocol design is to protect against *digital* attacks on integrity, anonymity and confidentiality. *Physical, economic and social* attacks are, and always will be, possible similarly to centralized systems (e.g. insider trading, cartels manipulating the underlying assets or the availability glitches such as the NASDAQ ones [38]) and they are typically dealt with by ex-post law enforcement [31].

2 Dark Pools as A Motivating Application For WKA

From a security perspective the constraints from the investor are easily captured by an NP-relation R as in Table 1 where the instance ϕ is the public information (i.e. the trader's commitment and the investor's constraints) and the witness ω is the private information (the trader's committed information). An investor may look for traders with enough capacity and use the Sufficient Capacity (SC) relation in Table 1. A trader may ask the investor to show interest in some price ranges, e.g. from p'_- to p'_+ using the Price Range (PR) relation and in addition check the consistency of the challenged threshold using Matchable Bid (MB), if the investor has previously committed to desired bid price p and volume v ,where $c' \geq pv$. Thus, the investor can simply post the relation R and use WKA

to securely agree on a secret key with each interested and eligible trader holding a secret witness ω (to their committed instance ϕ) that satisfies the desired relation, i.e. $R(\phi, \omega) = 1$. Each agreed key can then be used for the negotiation (usually a conversation, not just a single message) of the offer between the investor and each eligible trader.

Our WKA construction also aims for succinct communication which is important when using a distributed ledger. The committed information (the instance) is also frequently updated, while the relation R of interest may be persistent. WKA is advantageous in this case as it works efficiently with different instances of the same relation. Additionally, WKA allows the trader to send a message encrypted using the key along with the public response (that will be used by the verifier to reconstruct the key and decrypt the message). This may save one round and is key when executing over a blockchain.[6]

3 Related Work and Alternative Candidate Schemes

We summarize a comparison of WKA in terms of usability and efficiency against applicable alternative candidate schemes in Table. 2. (We refer the reader to the Appendix A of the full version of the paper [34] for more details).

A *trivial (but wrong)* solution is to ask each prover to couple a public key pk with a zk-SNARK proof $\boldsymbol{\pi}$ for the satisfaction of the arithmetic relation R. The verifier can then encrypt the private offer with pk after verifying the proof $\boldsymbol{\pi}$. Only the prover with the corresponding private key sk can decrypt. Since the decryption condition above says nothing about the validity of $\boldsymbol{\pi}$, one cannot guarantee that pk is actually from the prover that produced $\boldsymbol{\pi}$. Signature of Knowledge [25] (SoK), can be used to sign the public key pk. However, SoK delivers only pk of the prover thus allows only a one-way communication from the verifier to the prover. Further, the prover cannot make sure that the upcoming message encrypted with pk is from the verifier: as pk is public, anyone can see it and send a message to the prover using pk. Other similar generic constructions are generally based on the modification of R to include a transformation of k_r. Our WKA scheme uses directly R which yields a lower bound of circuit complexity. Besides, those approaches usually require full proof verification (that involves pairings, e.g. 5 as in [24]), which is more costly than our construction, where the Verifier directly forges the last proof element (only computation in the field F) and it even stops 1 step early.

[6] One can argue that there could be DDOS attacks where an attacker can post either malformed offers, or correctly formed ones but they have no intention of filling, to the blockchain. In the first case, as the Verifier only needs to forge the last proof element F (1) while the Prover has to compute the full proof (4(m-l+3n)) as shown in Table 3, such an attack will require tremendous effort from the Prover but not so much from the Verifier. In the second case, unfortunately we cannot solve this as it exists even in the centralized system. A trader/investor can post an offer, and cancel it before it is filled or immediately in the next round. However, at the point the offer was posted, the exchange cannot know whether the offer will be canceled or not.

Table 2. Comparison of Solutions

n is the number of parties. The comparison criteria include: (i) **A**: is anonymous communication supported? (ii) **PB**: does the solution satisfies proportional burdern, i.e. only the involved parties perform the computation? (iii) **DL**: does the solution considers the information bound on a distributed ledger? (iv) **AC**: are arithmetic circuits supported? (v) **BR**: blockchain-round complexity, i.e. the number of rounds happen on the blockchain; (vi) **BC**: blockchain-communication complexity, i.e. the size of the data communicated through the blockchain; and (vii) **C**: computational complexity.

Solution	**A**	**PB**	**DL**	**AC**	**BR**	**BC**	**C**
Full MPC [27]	y		y	y	13	$O(n^2)$	$O(n^2)$
2-3 Servers MPC [11]		y		y	N/A	N/A	$O(1)$
Paired 2PC [27]	y		y	y	2	$O(n)$	$O(n)$
Practical WE [18]	y	y	y		1	$O(1)$	$O(1)$
Practical AKE [26]	y	y	y		2	$O(1)$	$O(1)$
WKA (ours)	y	y	y	y	2	$O(1)$	$O(1)$

Full MPC fails proportional burden, yields an unacceptable 13 rounds of blockchain communication and have a high communication and computational complexity ($O(n^2)$). Using 2-3 servers MPC, one obtains better efficiency (no communication over blockchain; communication and computational complexity stay constant w.r.t n). Yet, it does not leverage existing ledgers; and anonymity, which is critical, is not guaranteed. Paired 2PC yields a desirable 2 rounds of blockchain communication. However, in order to guarantee anonymity, it fails proportional burden; the communication and computational complexity also become unacceptable ($O(n)$). Practical WE and AKE only supports algebraic relations. WKA is practical and satisfies all requirements.

Secure Multiparty Computation (MPC) [12] can be a general solution but is with either usability and efficiency issues. Firstly, setting up an MPC using existing distributed ledgers is not trivial as every party must be known in advanced or a PKI must be available in the setup phase for securing the communication over the ledger, e.g. as in [8]. Additionally, general Full MPC (where n parties join the computation, e.g. [27]) yields an unacceptable 13 rounds of blockchain communication; while the 2-3 Servers MPC (where n parties secret share their private inputs to the servers and let them perform the computation, e.g. [11]) and Paired 2PC (where the verifier contacts and perform a 2PC with each other party, e.g. [27]) fail to guarantee anonymity which can be critical [32].

Authenticated key exchanges (AKE) [5,9] only support relations on credentials. Here we have other relations among values not related to credentials as they can change dynamically. Language-AKE [26] is more flexible but it does not support non-algebraic relations such as SHA-256 employed by ZeroCash [36]. One can also use Witness Encryption [18] (WE) with the desired arithmetic relation R, and only the provers who possess the witness ω for that instance ϕ such that $R(\phi, \omega) = 1$ could decrypt. However, general WE constructions [16,20,17,3] are impractical while practical WE under a GS proof [13] cannot support arithmetic relation of depth greater than 1, e.g. SHA-256 as employed by ZeroCash [36]).

4 Witness Key Agreement

Notations A multivariate polynomial $t : \mathbb{F}^m \to \mathbb{F}$ over a finite field $\mathbb{F}$ has a degree d if the degree of each monomial in t is at most d and a monomial has degree d. A multivalued multivariate polynomial $\mathbf{t} : \mathbb{F}^m \to \mathbb{F}^\mu$ is a vector of polynomials $(t_1, \ldots, t_\mu)$ where each $t_i : \mathbb{F}^m \to \mathbb{F}$ is a multivariate polynomial. We denote a scalar by x and a vector by $\mathbf{x}$. We write $x \leftarrow \mathbb{X}$ when picking an element x uniformly from a finite set $\mathbb{X}$. We write $y \leftarrow \mathsf{A}(x)$ when picking the randomness r and returning $y = \mathsf{A}(x; r)$. $\Pr[\epsilon|\Omega]$ denotes the probability of an event ϵ over the probability space Ω. We denote the security parameter by 1^λ in the unary form and the negligible function as $\mathsf{negl}(\cdot)$. Given two probability functions $f, g : \mathbb{N} \to [0, 1]$ we write $f(\lambda) \approx g(\lambda)$ when $|f(\lambda) - g(\lambda)| = O(\lambda^{-c})$ for every constant $c > 0$. We say that f is *negligible* when $f(\lambda) \approx 0$.

Remark 1 (Generation of the relation R). We follow the notation of Groth [24] so that a relation generator $\mathcal{R}$ receives a security parameter 1^λ and returns a polynomial-time decidable binary relation R, i.e. $R \leftarrow \mathcal{R}(1^\lambda)$. Hence for notational simplicity we can assume 1^λ can be deduced from R.

Definition 1 (Witness Key Agreement). *Let L be an NP-language with the witness relation $R(\phi, \omega)$. We call ϕ an instance of L and ω a witness for ϕ. A Witness Key Agreement (WKA) scheme Ω for L is a tuple of polynomial-time algorithms (*$\mathsf{KChallenge}$*,* $\mathsf{KResponse}$*,* $\mathsf{KDerive}$*):*

$(\mathsf{p}_c, \mathsf{s}_c) \leftarrow \mathsf{KChallenge}(R)$ *is run by the verifier and takes as input the relation R (from which the security parameter 1^λ can be deduced), outputs a public and a secret challenge parameter $(\mathsf{p}_c, \mathsf{s}_c)$.*

$(\mathsf{p}_r, \mathsf{k}_r) \leftarrow \mathsf{KResponse}(R, \mathsf{p}_c, \phi, \omega)$ *is run by the prover with inputs the relation R, the public challenge parameter p_c, the instance ϕ, and the corresponding witness ω, outputs a public response parameter p_r and a secret key k_r.*

$\{\mathsf{k}_c, \perp\} \leftarrow \mathsf{KDerive}(R, \mathsf{s}_c, \phi, \mathsf{p}_r)$ *is run by the verifier and takes as input the relation R, the secret challenge parameter s_c, the instance $\phi$and the public response parameter p_r, outputs a key k_c or $\perp$.*

Security Properties WKA is closely related to Non-Interactive Zero-Knowledge (NIZK) Proof System. The key difference is the outcome of NIZK is only a binary verification result while WKA's outcome is a key upon success. Hence the security properties of WKA are also very similar to those of NIZK. Furthermore, we require WKA to be secure against MITM attack. (See Appendix B of [34] for a trivial WKA generic construction that is insecure under MITM attack.)

WKA Construction Roadmap We base our WKA construction on the efficient construction of zk-SNARK from Non-Interactive Linear Proof (NILP) [24] for Quadratic Arithmetic Programs (QAP) [19] given by Groth [24] and we utilize Linear-Only Encryption (LE) [6] to compile such NILP to a WKA scheme.

Linear Interactive Proofs (LIP) [6] is an extension of interactive proofs [23] in which each prover's message is an *affine combination* of the previous messages sent by the verifier.

Perfect Correctness Given a true instance, the key agreement is successful, i.e.

$$\Pr\left[\mathsf{k}_c = \mathsf{k}_r \;\middle|\; \begin{matrix} R \leftarrow \mathcal{R}(1^\lambda) \\ R(\phi,\omega) = 1 \end{matrix}, \begin{matrix} (\mathsf{p}_c, \mathsf{s}_c) \leftarrow \mathsf{KChallenge}(R) \\ (\mathsf{p}_r, \mathsf{k}_r) \leftarrow \mathsf{KResponse}(R, \mathsf{p}_c, \phi, \omega) \\ \mathsf{k}_c \leftarrow \mathsf{KDerive}(R, \mathsf{s}_c, \phi, \mathsf{p}_r) \end{matrix}\right] = 1 \tag{1}$$

Computational Adaptive Knowledge Soundness The key agreement is successful only with negligible probability if the prover knows no witness for the instance, i.e. for any PPT $\hat{\mathcal{A}}$, there exists a poly-time extractor $\epsilon_{\hat{\mathcal{A}}}$

$$\Pr\left[\begin{matrix} R(\phi,\omega) \neq 1 \\ \mathsf{k}_c = \mathsf{k}_r \end{matrix} \;\middle|\; \begin{matrix} R & \leftarrow \mathcal{R}(1^\lambda) \\ (\mathsf{p}_c, \mathsf{s}_c) & \leftarrow \mathsf{KChallenge}(R) \\ (\phi, \mathsf{p}_r, \mathsf{k}_r) & \leftarrow \hat{\mathcal{A}}(R, \mathsf{p}_c) \\ \mathsf{k}_c & \leftarrow \mathsf{KDerive}(R, \mathsf{s}_c, \phi, \mathsf{p}_r) \\ \omega & \leftarrow \epsilon_{\hat{\mathcal{A}}}(R, \phi, \mathsf{p}_r, \mathsf{k}_c) \end{matrix}\right] < \mathsf{negl}(\lambda) \tag{2}$$

Perfect Honest Verifier Zero-knowledge The response leaks nothing about the witness in the honest setup, i.e. there is a simulator $\mathcal{S}_{ZK}$ that outputs a simulated response $(\mathsf{p}_r, \mathsf{k}_r)$ and key k_c. Formally, for all $\lambda \in \mathbb{N}$, $R \leftarrow \mathcal{R}(1^\lambda)$, $R(\phi,\omega) = 1$ and any PPT $\hat{\mathcal{A}}$:

$$\begin{aligned} &\Pr\left[\hat{\mathcal{A}}(R, \mathsf{p}_c, \mathsf{s}_c, \phi, \mathsf{p}_r, \mathsf{k}_c) = 1 \;\middle|\; \begin{matrix} (\mathsf{p}_c, \mathsf{s}_c) \leftarrow \mathsf{KChallenge}(R) \\ (\mathsf{p}_r, \mathsf{k}_r) \leftarrow \mathsf{KResponse}(R, \mathsf{p}_c, \phi, \omega) \\ \mathsf{k}_c \leftarrow \mathsf{KDerive}(R, \mathsf{s}_c, \phi, \mathsf{p}_r) \end{matrix}\right] \\ &= \Pr\left[\hat{\mathcal{A}}(R, \mathsf{p}_c, \mathsf{s}_c, \phi, \mathsf{p}_r, \mathsf{k}_c) = 1 \;\middle|\; \begin{matrix} (\mathsf{p}_c, \mathsf{s}_c) \leftarrow \mathsf{KChallenge}(R) \\ (\mathsf{p}_r, \mathsf{k}_r, \mathsf{k}_c) \leftarrow \mathcal{S}_{ZK}(R, \mathsf{p}_c, \mathsf{s}_c, \phi) \end{matrix}\right] \end{aligned} \tag{3}$$

Perfect Response and Key Indistinguishability The public response and the agreed key can be simulated without knowledge of a witness, i.e. for all $\lambda \in \mathbb{N}$, $R \leftarrow \mathcal{R}(1^\lambda)$, $R(\phi,\omega) = 1$ there is a simulator $\mathcal{S}_{RKI}$ s.t. for any PPT $\hat{\mathcal{A}}$:

$$\begin{aligned} &\Pr\left[\hat{\mathcal{A}}(R, \mathsf{p}_c, \phi, \mathsf{p}_r, \mathsf{k}_r) = 1 \;\middle|\; \begin{matrix} (\mathsf{p}_c, \mathsf{s}_c) \leftarrow \mathsf{KChallenge}(R) \\ (\mathsf{p}_r, \mathsf{k}_r) \leftarrow \mathsf{KResponse}(R, \mathsf{p}_c, \phi, \omega) \end{matrix}\right] \\ &= \Pr\left[\hat{\mathcal{A}}(R, \mathsf{p}_c, \phi, \mathsf{p}_r, \mathsf{k}_r) = 1 \;\middle|\; \begin{matrix} (\mathsf{p}_c, \mathsf{s}_c) \leftarrow \mathsf{KChallenge}(R) \\ (\mathsf{p}_r, \mathsf{k}_r) \leftarrow \mathcal{S}_{RKI}(R, \mathsf{p}_c, \phi) \end{matrix}\right] \end{aligned} \tag{4}$$

Security against Man-In-The-Middle Attack The key agreement is successful only with negligible probability under Man-In-The-Middle Attack, i.e. for any PPT $\hat{\mathcal{A}}$:

$$\Pr\left[\mathsf{k}_c = \mathsf{k}'_r \;\middle|\; \begin{matrix} R \leftarrow \mathcal{R}(1^\lambda) \\ R(\phi,\omega) = 1 \end{matrix}, \begin{matrix} (\mathsf{p}_c, \mathsf{s}_c) \leftarrow \mathsf{KChallenge}(R) \\ (\mathsf{p}'_c, \mathsf{s}'_c) \leftarrow \mathsf{KChallenge}(R) \\ (\mathsf{p}_r, \mathsf{k}_r) \leftarrow \mathsf{KResponse}(R, \mathsf{p}'_c, \phi, \omega) \\ (\mathsf{p}'_r, \mathsf{k}'_r) \leftarrow \hat{\mathcal{A}}(R, \mathsf{p}_c, \mathsf{p}'_c, \mathsf{s}'_c, \mathsf{p}_r, \phi) \\ \mathsf{k}_c \leftarrow \mathsf{KDerive}(R, \mathsf{s}_c, \phi, \mathsf{p}'_r) \end{matrix}\right] < \mathsf{negl}(\lambda) \tag{5}$$

Fig. 1. Security of Witness Key Agreement Scheme

Groth renamed the input-oblivious two-message LIPs into NILP [24] to clarify the connection between LIP and NIZK. NILP considers only *adversaries using affine prover strategies*, i.e. a strategy which can be described by a tuple $(\boldsymbol{\Pi}, \boldsymbol{\pi}_0)$ where $\boldsymbol{\Pi} \in \mathbb{F}^{k \times y}$ represents a linear function and $\boldsymbol{\pi}_0 \in \mathbb{F}^k$ represents an affine shift. Then, on input a query vector $\boldsymbol{\sigma} \in \mathbb{F}^y$, the response vector $\boldsymbol{\pi} \in \mathbb{F}^k$ is constructed by evaluating the affine relation $\boldsymbol{\pi} = \boldsymbol{\Pi}\boldsymbol{\sigma} + \boldsymbol{\pi}_0$.

Key Observation. The proof $\boldsymbol{\pi}$ obtained with NILP consists of k elements (by evaluating k linear functions [7] corresponding to the proof matrix $\boldsymbol{\Pi}$), in which the k-th element can be obtained in two ways given the first $k-1$ elements [24]: (1) On the prover's side, if $\boldsymbol{\pi}$ is valid then the first $k-1$ elements fully determine the last one; (2) On the verifier's side, the first $k-1$ elements can be used in a proof forging formula to obtain the last one. By the prover computing $\boldsymbol{\pi}$ and publishing the first $k-1$ elements of $\boldsymbol{\pi}$, both parties can agree on the last element to use as a shared secret key for secure communication.[8] With this observation we construct WKA from a new NILP notion: *split designated verifier NILP*. (§5).

Succinct zero-knowledge non-interactive argument of knowledge (zk-SNARK) follows the relaxation from Perfect Soundness to Computational Soundness [21]. Bitansky *et al.* [6] also showed that NILP can be compiled into both publicly verifiable (verifier degree 2, using bilinear maps) and designated-verifier (using linear-only encryption scheme) zk-SNARK. Intuitively the prover computes the proof $\boldsymbol{\pi}$ as linear combinations of the CRS $\boldsymbol{\sigma}$ and the verifier checks the argument by checking the quadratic equations corresponding to the relation R.

Linear-Only Encryption (LE) scheme Σ (Bitansky *et al.* [6]), e.g. a two-ciphertexts variant of Paillier [35], is a tuple of polynomial-time algorithms (KeyGen, Enc, ImgVer, Dec, Add) where the ImgVer (image verification) prevents oblivious ciphertext samplings in the image of Enc using pk, i.e. this property prevents the adversary from encrypting plaintexts from scratch (see appendix E of [34] for further details), and Add is for evaluating linear combinations of valid ciphertexts. An LE scheme satisfies *correctness*, *additive homomorphism*, *indistinguishability under chosen plaintext attack* (IND-CPA) and in addition *linear-only homomorphism* which essentially says that it is infeasible to generate a new valid ciphertext except by evaluating an affine combination of valid ciphertexts (via Add)[9]. Such LE scheme can be instantiated using existing encryption schemes. The security of an LE scheme relies on the assumptions of q-power Diffie-Hellman, q-power Knowledge of Exponent and q-power Knowledge of Equality [6].

[7] In the concrete construction by Groth [24] (see also Fig. 3), $k = 3$ and the proof matrix $\boldsymbol{\Pi}$ is represented as the coefficients of the linear functions.

[8] The concrete example of this observation can be seen in Fig. 3 in section §6. The first two elements A and B (Eq. (7) and (8)) uniquely define C (Eq. (9)) and they can be fed into the proof forging formula (Eq. (11)) to get the 3rd element C which should be the same for either party.

[9] This property formally guarantees that given a valid ciphertext $\boldsymbol{\pi}$ by an adversary, it is possible to *efficiently* extract the corresponding affine function $(\boldsymbol{\Pi}, \boldsymbol{\pi}_0)$ that explains $\boldsymbol{\pi}$. Such property is important for Knowledge Soundness of WKA.

For relation functionality and efficiency in WKA we leverage on *Quadratic Arithmetic Programs* (QAP) by Gennaro et. al. [19]: an arithmetic circuit can be transformed into a system of equations that check the consistency of a set of instance variables ϕ and witness variables ω in a relation R. The consistency checker is compiled into zk-SNARK. Thus zk-SNARK for QAP covers applications that employ arithmetic relations of multiplicative depth larger than one such as SHA256. In our WKA construction the partial proof size is also succinct, as it has at most 3 elements regardless of R. Response computation and key derivation are efficient, i.e. only linear in QAP size.

Limitations of our WKA construction Our WKA scheme, as any scheme, inherits the limitations of its components:, i.e. the designated-verifier zk-SNARK that is compiled from an NILP for QAP by Groth [24]. Firstly zk-SNARKs are not known to satisfy composability and therefore cannot be run out of the box in parallel in the design of larger protocols [30].[10] In a basic dark pool scenario we only consider sequential composition where each execution of WKA concludes before the next execution begins [10]. For extended scenario one might need to use other instruments to identify parallel runs as described in Principle 10 of security protocol design by Abadi and Needham [1]. However, note that we still consider security against MITM attack, which is important for key agreement protocols. Secondly our WKA scheme makes use of QAP [19] hence it is only as efficient as the circuit expressing the constraints. Finally, we opted for simplicity rather than making the WKA scheme subversion-resistant as this which would require the zero-knowledge property be maintained even when the CRS is maliciously generated (see Bellare *et al.* [4]). Abdolmaleki *et al.* [2] and Fuchsbauer [15] constructed subversion-resistant NIZK based on Groth's zk-SNARK construction [24]. However, both works consider only the publicly verifable zk-SNARK construction based on bilinear groups. Our WKA construction requires designated-verifier zk-SNARK, and therefore those constructions are not applicable to our scheme. Hence, we consider only honest setups.[11]

5 WKA From NILP

We first define our *split designated verifier NILP* based on Groth's definition [24]. The CRS is first split into two parts $(\boldsymbol{\sigma}_P, \boldsymbol{\sigma}_V)$ where $\boldsymbol{\sigma}_V$ is only available to the verifier. Subsequently, in proof computation we split the proof matrix $\boldsymbol{\Pi} \in \mathbb{F}^{k \times y}$ into two parts: $\boldsymbol{\Pi}_1 \in \mathbb{F}^{k-1 \times y}$ and $\boldsymbol{\Pi}_2 \in \mathbb{F}^{1 \times y}$. The proof $\boldsymbol{\pi}$ is also split into

[10] Users are advised to run the shared secret through a hash function modelled as a random oracle before using it as a key for any other cryptosystem.

[11] Such an assumption can be relaxed by asking a TTP to generate the CRS (such as Bloomberg itself). Using a TTP for bootstrapping security protocols have been considered in literature, see for example HAWK [29]. This is a much weaker trust assumption than managing orders themselves because the generation of the CRS requires only the relation R and the public key for the encryption. Therefore such a TTP is only trusted to do the computation correctly. Without the private key, the TTP cannot learn additional information.

$\boldsymbol{\pi}_1 = \boldsymbol{\Pi}_1\boldsymbol{\sigma}_P$ that consists of $k-1$ elements and $\boldsymbol{\pi}_2 = \boldsymbol{\Pi}_2\boldsymbol{\sigma}_P$ consists of the last element. This split of $\boldsymbol{\Pi}$ and $\boldsymbol{\pi}$ is not necessary in a zk-SNARK proof system but it is essential in our WKA scheme as we need to split the proof into two parts (See our key observation in §4).

Definition 2 (Split designated-verifier NILP). *Let L be an NP-language with the witness relation $R(\phi, \omega)$. We call ϕ an instance of L and ω a witness for ϕ. A split designated-verifier (split DV) NILP for L consists of the tuple of polynomial-time algorithms (*Setup*,* Prove*,* Verify*,* $\mathsf{Simulate}$*):*

$(\boldsymbol{\sigma}_P, \boldsymbol{\sigma}_V) \leftarrow \mathsf{Setup}(R)$**:** *output $\boldsymbol{\sigma}_P \in \mathbb{F}^y$ and $\boldsymbol{\sigma}_V \in \mathbb{F}^x$.*
$(\boldsymbol{\pi}_1, \boldsymbol{\pi}_2) \leftarrow \mathsf{Prove}(R, \boldsymbol{\sigma}, \phi, \omega)$**:** *obtain $(\boldsymbol{\Pi}_1, \boldsymbol{\Pi}_2) \leftarrow \mathsf{ProofMatrix}(R, \phi, \omega)$ where $\boldsymbol{\Pi}_1 \in \mathbb{F}^{k-1\times y}$ and $\boldsymbol{\Pi}_2 \in \mathbb{F}^{1\times y}$ and output $\boldsymbol{\pi}_1 = \boldsymbol{\Pi}_1\boldsymbol{\sigma}_P$ and $\boldsymbol{\pi}_2 = \boldsymbol{\Pi}_2\boldsymbol{\sigma}_P$*
$\{0,1\} \leftarrow \mathsf{Verify}(R, \boldsymbol{\sigma}_V, \phi, \boldsymbol{\pi}_1, \boldsymbol{\pi}_2)$**:** *obtain $\mathbf{t} \leftarrow \mathsf{Test}(R, \phi)$ where $\mathbf{t} : \mathbb{F}^{y+k} \to \mathbb{F}^{\eta}$ is an arithmetic circuit corresponding to the evaluation of multivariate polynomials such that $\mathbf{t}(\boldsymbol{\sigma}_V, \boldsymbol{\pi}_1, \boldsymbol{\pi}_2) = 0$ if $\boldsymbol{\pi}$ is valid..*
$(\boldsymbol{\pi}_1, \boldsymbol{\pi}_2) \leftarrow \mathsf{Simulate}(R, \boldsymbol{\sigma}_V, \phi)$**:** *obtain $\mathbf{t} \leftarrow \mathsf{Test}(R, \phi)$ and solve $\mathbf{t}(\boldsymbol{\sigma}_V, \boldsymbol{\pi}_1, \boldsymbol{\pi}_2) = 0$ for the output $(\boldsymbol{\pi}_1, \boldsymbol{\pi}_2)$.*

where y, x, k, η and d are constants or polynomials in 1^λ (deduced from R [24]).

A tuple of PPT algorithms (Setup, Prove, Verify, $\mathsf{Simulate}$) is a split DV NILP if it has perfect completeness, perfect zero-knowledge and statistical soundness against affine prover strategies.

Construction of Witness Key Agreement We construct WKA from Split DV NILP as shown in Fig. 2. Below we describe the construction at a high level.

We first modify the LE scheme's encryption algorithm interface for explicit used randomness. We omit the randomness r and write only $[m] \leftarrow \mathsf{Enc}(\mathsf{pk}, m)$ in case r is not necessary in subsequent computation. We write $[m] = \mathsf{Enc}(\mathsf{pk}, m, r)$ to incorporate the randomness directly into the encryption algorithm. Secondly we require that the additive homomorphism of LE applies to both the message and the randomness used, i.e. $\mathsf{Add}(\mathsf{pk}, \langle [m_i, r_i] \rangle, \langle \alpha_i \rangle)$ evaluates $[\sum \alpha_i m_i, \sum \alpha_i r_i]$.

The challenge phase. In $\mathsf{KChallenge}$, the verifier generates a CRS ($\boldsymbol{\sigma}_P$, $\boldsymbol{\sigma}_V$) from R (using a split DV NILP). The verifier then encrypts each elements $\{\sigma_{P,i}\}_{i=1}^{y}$ of the $\boldsymbol{\sigma}_P$ with an LE scheme (with key pair pk, sk). Additionally, we require the verifier to encrypt the randomnesses $\{r_{P,i}\}_{i=1}^{y}$ that are used for the encryption of the CRS $\{\sigma_{P,i}\}_{i=1}^{y}$ in $\mathsf{KChallenge}$ into $\{[r_{P,i}]\}_{i=1}^{y}$. Finally s/he publishes a challenge that consists of pk and the encrypted elements. The verifier keeps private sk of the LE scheme and the plain CRS $\boldsymbol{\sigma}_V$.

The response phase. Upon seeing the challenge, in $\mathsf{KResponse}$, the prover computes a response by generating a valid proof $\boldsymbol{\pi}$ for the desired tuple (ϕ, ω) (using the proof matrix of the split DV NILP and the additive homomorphic operation Add of the LE scheme). When the prover evaluates the last encrypted element $[\pi_2, r_2]$ using the proof matrix $\boldsymbol{\Pi}_2$ and the encrypted CRS $\{[\sigma_{P,i}]\}_{i=1}^{y}$, by the additively homomorphic property of the LE scheme, s/he can also evaluate the ciphertext $[r_2]$ of the randomness r_2 of the encrypted $[\pi_2, r_2]$ using the same $\boldsymbol{\Pi}_2$ and $\{[r_{P,i}]\}$. The prover publishes the first encrypted $k-1$ elements

Following Groth [24] we assume 1^λ can be deduced from R.

$(\mathsf{p}_c, \mathsf{s}_c) \leftarrow \mathsf{KChallenge}(R)$ runs as follows.

1. Fix a linear-only encryption scheme Σ;
2. Run $(\mathsf{pk}, \mathsf{sk}) \leftarrow \Sigma.\mathsf{KeyGen}(1^\lambda)$ where 1^λ is the security parameter deduced from R (see Remark 1); and $(\boldsymbol{\sigma}_P, \boldsymbol{\sigma}_V) \leftarrow \mathsf{Setup}(R)$;
3. Encrypt $[\sigma_{P,i}, r_{P,i}] \leftarrow \Sigma.\mathsf{Enc}(\mathsf{pk}, \sigma_{P,i})$ for each $\sigma_{P,i} \in \boldsymbol{\sigma}_P$;
4. Encrypt $[r_{P,i}] \leftarrow \Sigma.\mathsf{Enc}(\mathsf{pk}, r_{P,i})$ for each $r_{P,i}$ above;
5. Return $\mathsf{p}_c = (\mathsf{pk}, \{[\sigma_{P,i}, r_{P,i}]\}_{i=1}^{y}, \{[r_{P,i}]\}_{i=1}^{y})$ and $\mathsf{s}_c = (\mathsf{sk}, \boldsymbol{\sigma}_V)$.

$(\mathsf{p}_r, \mathsf{k}_r) \leftarrow \mathsf{KResponse}(R, \mathsf{p}_c, \phi, \omega)$**:** Upon receiving the challenge p_c,

1. Run $(\boldsymbol{\Pi}_1, \boldsymbol{\Pi}_2) \leftarrow \mathsf{ProofMatrix}(\phi, \omega | R)$;
2. Compute $\{[\pi_{1,j}, r_{1,j}]\}_{j=1}^{k-1} = \boldsymbol{\Pi}_1(\{[\sigma_{P,i}, r_{P,i}]\}_{i=1}^{y})$ (with $\Sigma.\mathsf{Add}$);
3. Compute $[\pi_2, r_2] = \boldsymbol{\Pi}_2(\{[\sigma_{P,i}, r_{P,i}]\}_{i=1}^{y})$ (with $\Sigma.\mathsf{Add}$);
4. Compute $[r_2] = \boldsymbol{\Pi}_2(\{[r_{P,i}]\}_{i=1}^{y})$ (with $\Sigma.\mathsf{Add}$);
5. Return $\mathsf{p}_r = (\{[\pi_{1,j}, r_{1,j}]\}_{j=1}^{k}, [r_2])$ and $\mathsf{k}_r = [\pi_2, r_2]$.

$\{\mathsf{k}_c, \perp\} \leftarrow \mathsf{KDerive}(R, \mathsf{s}_c, \phi, \mathsf{p}_r)$ Output $\perp$ if any verification fails:

1. Verify $\mathsf{ImgVer}(\mathsf{sk}, [\pi_{1,j}, r_{1,j}]) = 1$ for $1 \le j \le k-1$; and $\mathsf{ImgVer}(\mathsf{sk}, [r_2]) = 1$;
2. Decrypt $\pi_{1,j} = \Sigma.\mathsf{Dec}(\mathsf{sk}, [\pi_{1,j}, r_{1,j}])$ for $1 \le j \le k-1$;
3. Obtain $\mathbf{t} \leftarrow \mathsf{Test}(R, \phi)$; use $\{\pi_{1,j}\}_{j=1}^{k-1}$ to solve $\mathbf{t}(\boldsymbol{\sigma}_V, \{\pi_{1,j}\}_{j=1}^{k}, \pi_2) = 0$ for π_2;
4. Decrypt $r_2 = \Sigma.\mathsf{Dec}(\mathsf{sk}, [r_2])$
5. Return $\mathsf{k}_r = \Sigma.\mathsf{Enc}(\mathsf{pk}, \pi_2, r_2)$ (r_2 as randomness).

Fig. 2. Construction of Witness Key Agreement

$\{[\pi_{1,j}, r_{1,j}]\}_{j=1}^{k-1}$ and the encrypted randomness $[r_2]$ as a public response and keeps secret the last encrypted element $[\pi_2, r_2]$.

The key derive phase. When the verifier sees the instance ϕ and the corresponding response, in $\mathsf{KDerive}$, s/he can decrypt the encrypted elements using sk to get $\{\pi_{1,j}\}_{j=1}^{k-1}$ and forge the last element π_2 using the plain CRS $\boldsymbol{\sigma}_V$. The verifier then uses the evaluated $[r_2]$ to reconstruct the correct ciphertext $[\pi_2, r_2]$ of the last element, i.e. the verifier decrypts $[r_2]$ to get r_2 to use as the randomness in the final encryption of π_2 to get $[\pi_2]$. After that, both parties agree on the same $[\pi_2, r_2]$.

We refer the reader to Appendix C of [34] for the proof sketch of our main theorem as follows.

Theorem 1 (Security of WKA). *If Σ satisfies correctness, additive homomorphism, IND-CPA and linear-only homomorphism, and the underlying split DV NILP satisfies perfect completeness, perfect zero-knowledge and statistical knowledge soundness against affine prover strategies, then Ω satisfies correctness, adaptive knowledge soundness, honest verifier zero-knowledge, response and key indistinguishability, and security against man-in-the-middle attack.*

6 WKA from NILP based on QAP

We recall the formal definition of Quadratic Arithmetic Programs (QAP) [19] and how to construct a NILP for QAP [24].

We consider the QAP that defines a binary relation R as described in Remark 2. NILP for such QAP is defined as a tuple of polynomial-time algorithms (Setup, Prove, Verify, Simulate):

$(\boldsymbol{\sigma}, \boldsymbol{\tau}) \leftarrow \mathsf{Setup}(R)$**:** Pick $\alpha, \beta, \gamma, \delta, x \leftarrow \mathbb{F}^*$. Set $\boldsymbol{\tau} = (\alpha, \beta, \gamma, \delta, x)$ and $\boldsymbol{\sigma}$:

$$\begin{aligned} \boldsymbol{\sigma} = \alpha, \beta, \gamma, \delta, \{x^i\}_{i=0}^{n-1}, \left\{ \frac{\beta u_i(x) + \alpha v_i(x) + w_i(x)}{\gamma} \right\}_{i=0}^{l}, \\ \left\{ \frac{\beta u_i(x) + \alpha v_i(x) + w_i(x)}{\delta} \right\}_{i=l+1}^{m}, \left\{ \frac{x^i t(x)}{\delta} \right\}_{i=0}^{n-2} \end{aligned} \tag{6}$$

$\boldsymbol{\pi} \leftarrow \mathsf{Prove}(R, \boldsymbol{\sigma}, a_1, \ldots, a_m)$**:** Pick $r, s \leftarrow \mathbb{F}$ and compute

$$A = \alpha + \sum_{i=0}^{m} a_i u_i(x) + r\delta \tag{7}$$

$$B = \beta + \sum_{i=0}^{m} a_i v_i(x) + s\delta \tag{8}$$

$$C = \sum_{i=l+1}^{m} a_i \frac{\beta u_i(x) + \alpha v_i(x) + w_i(x)}{\delta} + \frac{h(x)t(x)}{\delta} + sA + rB - rs\delta \tag{9}$$

In NILP [24], $\boldsymbol{\pi} = (A, B, C)$. In Split DV NILP, $\boldsymbol{\pi}_1 = (A, B)$ and $\boldsymbol{\pi}_2 = (C)$.
$\{0, 1\} \leftarrow \mathsf{Verify}(R, \boldsymbol{\sigma}, a_1, \ldots, a_l, \pi)$**:** Output 1 iff:

$$AB = \alpha\beta + \sum_{i=0}^{l} a_i \frac{\beta u_i(x) + \alpha v_i(x) + w_i(x)}{\gamma}\gamma + C\delta \tag{10}$$

$\boldsymbol{\pi} \leftarrow \mathsf{Simulate}(\boldsymbol{\tau}|R, a_1, \ldots, a_l)$**:** Pick $A, B \leftarrow \mathbb{F}$, and output $\boldsymbol{\pi} = (A, B, C)$ where:

$$C = \frac{AB}{\delta} - \frac{\alpha\beta}{\delta} - \frac{\sum_{i=0}^{l} a_i(\beta u_i(x) + \alpha v_i(x) + w_i(x))}{\delta} \tag{11}$$

Fig. 3. Split NILP for QAP based on Groth [24])

Definition 3 (QAP). *A quadratic arithmetic program $\mathbb{Q}$ over a field $\mathbb{F}$ for a relation $R(\phi, \omega)$ consists of three sets of polynomial $\{u_i(X), v_i(X), w_i(X)\}_{i=0}^{m}$ and a target polynomial $t(X) = \Pi_{q=1}^{n}(X - r_q)$ such that with $a_0 = 1$, $\phi = \{a_i\}_{i=1}^{l}$, and $\omega = \{a_i\}_{i=l+1}^{m}$, the following Eq.* (12) *holds.*

$$\sum_{i=0}^{m} a_i u_i(X) \sum_{i=0}^{m} a_i v_i(X) = \sum_{i=0}^{m} a_i w_i(X) + h(X)t(X) \tag{12}$$

where $u_i(X), v_i(X), w_i(X)$ are of degee $n-1$ and $h(X)$ is of degree $n-2$.

Remark 2 (QAP description). For convenience we follow the QAP description of Groth [24], we consider the QAP

We assume 1^λ can be deduced from R. Comparing to the original NILP in Fig. 3, our NILP does not make use of γ. As we only need Eq (7), (8), (9) and (11) that do not contains γ (γ is only needed in the verification equation Eq. (10)).

$\underline{(\mathsf{p}_c, \mathsf{s}_c) \leftarrow \mathsf{KChallenge}(R)}$**:** Fix an LE scheme $\mathbf{\Sigma}$ (with key pair $(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathbf{\Sigma}.\mathsf{KeyGen}(1^\lambda)$), run $(\boldsymbol{\sigma}_P, \boldsymbol{\sigma}_V) \leftarrow \mathsf{Setup}(R)$ to obtain $\boldsymbol{\sigma}_V = (\alpha, \beta, \delta, x)$ and generate $\{[\sigma_{P,i}, r_{P,i}] \leftarrow \mathbf{\Sigma}.\mathsf{Enc}(\mathsf{pk}, \sigma_{P,i})\}$ and $[r_{P,i}] \leftarrow \mathbf{\Sigma}.\mathsf{Enc}(\mathsf{pk}, r_{P,i})$ for each $\sigma_{P,i} \in \boldsymbol{\sigma}_P$ where

$$\boldsymbol{\sigma}_P = \alpha, \beta, \delta, \{x^i\}_{i=0}^{n-1}, \left\{\frac{\beta u_i(x) + \alpha v_i(x) + w_i(x)}{\delta}\right\}_{i=l+1}^{m}, \left\{\frac{x^i t(x)}{\delta}\right\}_{i=0}^{n-2}; \quad (13)$$

Return $\mathsf{p}_c = (\mathsf{pk}, \{[\sigma_{P,i}, r_{P,i}]_{i=1}^{y}\}, \{[r_{P,i}]\}_{i=1}^{y})$ and $\mathsf{s}_c = (\mathsf{sk}, \boldsymbol{\sigma}_V)$.

$\underline{(\mathsf{p}_r, \mathsf{k}_r) \leftarrow \mathsf{KResponse}(\phi = \{a_i\}_{i=0}^{l}, \omega = \{a_i\}_{i=l+1}^{m}, R, \mathsf{p}_c)}$**:** Upon receiving the challenge p_c,

1. Pick $r, s \leftarrow \mathbb{F}$;
2. Compute $[A]$, $[B]$, and $[C]$ (as well as $[r_2]$) using the affine functions in Fig. 3 (Eq. (7), (8) and (9)) on $\{[\sigma_{P,i}, r_{P,i}]_{i=1}^{y}\}$ (and $\{[r_{P,i}]\}_{i=1}^{y}$) with $\mathbf{\Sigma}.\mathsf{Add}$;
3. Set $[\pi_{1,1}] = [A]$, $[\pi_{1,2}] = [B]$ and $[\pi_2, r_2] = [C]$;
4. Return $\mathsf{p}_r = ([\pi_{1,1}], [\pi_{1,2}], [r_2])$ and $\mathsf{k}_r = [\pi_2, r_2]$.

$\underline{\{\mathsf{k}_c, \perp\} \leftarrow \mathsf{KDerive}(R, \mathsf{s}_c, \phi, \mathsf{p}_r)}$ outputs $\perp$ if any verification fails:

1. Verify $\mathsf{ImgVer}(\mathsf{pk}, [\pi_{1,j}]) = 1$ for $j = \{1, 2\}$;
2. Verify $\mathsf{ImgVer}(\mathsf{pk}, [r_2]) = 1$;
3. Decrypt $A = \mathbf{\Sigma}.\mathsf{Dec}(\mathsf{sk}, [\pi_{1,1}])$; and $B = \mathbf{\Sigma}.\mathsf{Dec}(\mathsf{sk}, [\pi_{1,2}])$;
4. Decrypt $r_2 = \mathbf{\Sigma}.\mathsf{Dec}(\mathsf{sk}, [r_2])$;
5. Compute C as in Eq. (11) with A and B;
6. Return $\mathsf{k}_r = \mathbf{\Sigma}.\mathsf{Enc}(\mathsf{pk}, C, r_2)$ (using r_2 as randomness).

Fig. 4. Witness key agreement for QAP

R, i.e.

$$(\mathbb{F}, aux, l, \{u_i(X), v_i(X), w_i(X)\}_{i=0}^{m}, t(X))$$

where $\mathbb{F}$ is a finite field; aux is some auxiliary information; $1 \leq l \leq m$; $u_i(X)$,$v_i(X)$,$w_i(X)$, $t(X) \in \mathbb{F}[X]$, $u_i(X)$,$v_i(X)$,$w_i(X)$ are of at most degree $n-1$. Such QAP defines a binary relation

$$R = \left\{ (\phi, \omega) \middle| \begin{array}{c} a_0 = 1, \phi = \{a_i\}_{i=1}^{l}, \omega = \{a_i\}_{i=l+1}^{m} \\ \sum_{i=0}^{m} a_i u_i(X) \sum_{i=0}^{m} a_i v_i(X) \\ = \sum_{i=0}^{m} a_i w_i(X) + h(X)t(X) \end{array} \right\}$$

A split DV NILP for QAP can be directly reformulated as in Fig. 3 by modifying the Prove algorithm. We simply split the proof matrices into two matrices $\boldsymbol{\Pi}_1$ and $\boldsymbol{\Pi}_2$ where $\boldsymbol{\Pi}_1 \in \mathbb{F}^{2\times y}$ corresponds to the matrix used in Eq. (7) and (8) while $\boldsymbol{\Pi}_2 \in \mathbb{F}^{1\times y}$ corresponds to the matrix used in Eq. (9). Since the NILP in Fig. 3 is secure (see Groth's security proof [24, Theorem 1]), our split DV NILP is also secure (see Appendix D of [34]). We show in Fig. 4 how to construct Ω using a split DV NILP obtained from the NILP in Fig. 3.

Table 3. Theoretical Performance Evaluation

Alg.	#Enc	#Dec	#Mult
KChallenge	$4(m-l+2n)$	-	-
KResponse	-	-	$4(m-l+3n)$
KDerive	1	k	-

m is the number of variables in a QAP, l is the number of instance variables, and $n-1$ is the degree of polynomials in the QAP. The number of decryption k is construction dependent. In our case we have $k = 3$.

Theorem 2. *If the LE scheme Σ satisfies correctness, additive homomorphism, IND-CPA and linear-only homomorphism, then the construction in Fig. 4 yields a WKA scheme Ω that satisfies correctness, adaptive knowledge soundness, honest verifier zero-knowledge, response and key indistinguishability, and security against man-in-the-middle attack.*

7 Instantiation And Performance Evaluation

Instantiation We choose to instantiate the linear-only encryption scheme Σ with a variant of the Paillier cryptosystem [35] similarly to Gennaro *et al.* [19] and Bitansky *et al.* [6] (see Appendix E of [34]).

Theoretical WKA Performance Evaluation We can then estimate the theoretical performance of our WKA scheme Ω based on the number of encryptions, decryptions, and scalar multiplications for computing $\boldsymbol{\Pi}_1(\{[\sigma_{P,i}]\})$ and $\boldsymbol{\Pi}_2(\{[\sigma_{P,i}]\})$ (Table. 3). Let m be the number of variables of a QAP, l be the number of instance variables, and $n-1$ be the degree of polynomials of the QAP. The KChallenge algorithm requires the generation of $\{[\sigma_{P,i}]\}$ hence $m-l+2n$ encryptions on the investor's side. The KResponse algorithm requires only the proof computation on the trader's side which yields $m-l+3n$ scalar multiplications. The above numbers are doubled to fix the malleability of the scheme (see Appendix E of [34]). It is then doubled again for computing the ciphertexts of the randomnesses. Finally the KDerive algorithm only requires k decryptions and one encryption on the investor's side. The proof size is also only 6 Paillier ciphertexts.

Baseline Performance Paillier [35] is the main ingredient in our construction and its performance is well-studied in literature. Several optimization techniques were already present in the original paper [35], and Jost *et al.* [28] took a step further to improve the performance by orders of magnitude faster compared to a naïve implementation.

For the timing of the Paillier encryption scheme we use the data from Table 4 by Jost *et al.* [28] *as an upper bound*[12] for the encryption time. The numbers were obtained on an Intel i7-4600U CPU at 2.10GHz with 4 cores running Ubuntu 64-bit v14.04. In particular, the reported result shows that, at 2048-bit key length,

[12] Benchmarked in 2015. As such, it provides a lower bound to our WKA performance

Table 4. Specific circuit evaluation

Relation R	m	$n-1$	$\mathcal{T}_C$ (s)	$\mathcal{T}_R$ (s)
SC	25821	28312	5.8	7.8
PR	26080	28572	5.8	7.9
PR'	26598	29094	6	8
MB	51382	56361	11.6	15.6

We support 2048-bit key length and provide 112-bit security. Recall m is the number of variables and $n-1$ is the degree of polynomials of the QAP. SC and PR are used for our dark pool simulation.

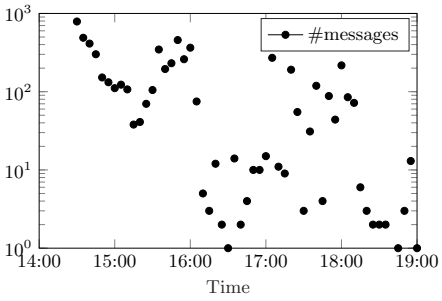
the encryption rate for 32-bit messages can reach 56K/s at the cost of 5.7s precomputation time.

Circuit Evaluation We implement the relations SC, PR, MB and a new relation PR' which is the same as PR but with added check, e.g. $(p_1 < p < p_2) \vee (p_3 < p < p_4)$, in Table 1 as arithmetic circuits with the libsnark library [37] and measure the number of required variables m and the corresponding degree of the polynomials $(n-1)$. Finally the runtime of KChallenge and KResponse, the most costly for 138-bit security for guessing r [28], 2048-bit key length, using the 32-bit messages and the encryption rate as in Scheme 3 from Jost *et al.* [28]. The evaluation of the new PR' relation and the MB relation illustrates the scalability of WKA. PR' consists of 1 consistency check for 1 commitment (1 private variable) and 4 arithmetic conditions with public variables, while MB consists of 2 consistency checks for 2 commitments (2 private variables). MB is in fact a building block for more general relation: $c' > p_1 \cdot v_1 + p_2 \cdot v_2 + p_3 \cdot p_3 + \ldots p_h \cdot v_h$. This is usable for both Multi-bids Auction and Biometrics Sharing (Hamming distance between two extracted features). This will require $2h$ commitments as it scales linearly with the number of private variables.

As shown in Table 4, the performances of SC, PR and PR' are close as their circuit complexity are similar to each other, as SC, PR and PR' require only one commitment consistency check while MB requires two of them. Hence, the runtime of MB is approximately double that of the others. KChallenge ($\mathcal{T}_C$) requires only 5.8s for the SC while PI takes only 5.9s. After the KChallenge, the key-agreement with KResponse ($\mathcal{T}_R$) takes only 7.8s for SC and 7.9s for PR. Even if we add 1s of one-way network latency into each message as we are employing an anonymous network (e.g. Tor) [14, Fig. 2]. The overhead of each WKA operation is lower than any known permission-less blockchain's block generation time (with Ethereum being the fastest at around 15s).[13] Hence each step can be fit within a single block generation time.[14]

[13] `https://ethstats.net/`.

[14] In our protocol, the blockchain is the actual bottleneck. Looking at Table 4, the runtime of each step (including setups) is less than the block time of the fastest permissionless blockchain (Ethereum roughly generates a block every 15s). Hence evaluating the interfaces of our scheme with the blockchain is equivalent to evaluating the blockchain itself. We should add that the current blockchain technologies is not adequate yet for high speed dark pools. Our major concern and main evaluation focus therefore is our scheme's crypto overhead.



For an actual trade it requires multiple messages. For a high day, the number of messages can reach 14103 (with 55 trades). For a low day, the number of messages is only 4514 (with 53 trades).

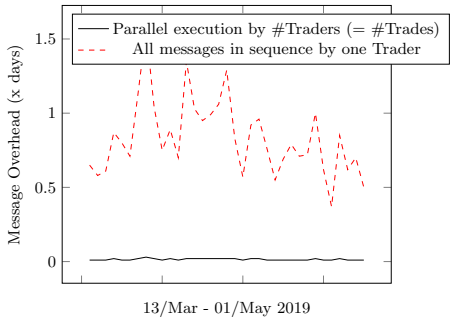
Fig. 5. Example of Tradebook messages and trades (May 1st, 2019)

Dark Pools Simulation For our simulation we make use of the Bloomberg Tradebook [7] for the period 13/03-1/5/2019 (35 trading days).

The Tradebook only contains the number of messages and the number of trades per day (see Fig. 5). Using WKA, an investor can setup a secure conversation including multiple messages which eventually lead to a trade. This means that the number of conversations (i.e. the truly necessary WKA executions) can be much smaller than the number of messages in Fig. 5. These conversations can also happen in parallel if they belong to different trades (or traders). From the available data we cannot know exactly *which messages belong to the same conversation*, or *how many conversations there are* and *the point of time at which they happened* as this is the whole point of a Dark Pool. We therefore considered the *worst possible case* where each message is a conversation by itself (almost always ending nowhere) and they are executed sequentially one after another by a single trader. We also considered a more plausible scenario *one trade-one trader* where each trade is done by a different trader and all messages of the day eventually belong to some trade.

We can combine the number of messages and trades from the extracted market data (examples shown in Fig. 5) and Table 4 to estimate the corresponding execution overhead throughout a day of trading. The final results are reported in Fig. 6. Performance is evaluated in terms of execution overhead to the expected processing time (1 day) as in a realistic setting using actual trading data is at least comparable on a day by day basis: if we were to run a day of trading messages, we would expect it to not take more than a day to actually exchange those messages.

We combine the relations SC with PR and we consider the execution time of a message as the running time of SC's KChallenge (5.8s). For trades execution time we consider the sequential execution of KResponse from SC and the whole challenge and response time of PR (21.6s), adding the one-way delay of Tor (1s) per message. As shown in Fig. 6, even under worst possible assumption, only 7 days out of 35 days require more than 1 day of execution in our simulation. With a less extreme approach (solid line) the overhead is smaller than 10%.



Assuming all computation done sequentially by 1 trader and messages are sent through Tor, only 7 out of 35 days of trading exhibits overheads greater than 1x in our simulation. With a more plausible scenario (each trade is done by a different trader) the overhead is at most 2%.

Fig. 6. WKA Evaluation on Bloomberg Tradebook

8 Conclusion

We introduced the notion of witness-key-agreement. Specifically we defined split designated-verifier non-interactive linear proof following Groth's definition of NILP [24]. We then compiled the obtained split DV NILP into a Witness Key Agreement scheme using Linear-Only Encryption. Our obtained construction is efficient. After a one-time setup that yields a common challenge for a relation R of interest, a party can agree on a secret key with another party given that the latter knows a witness of a committed instance.

Finally, our concrete WKA scheme for quadratic arithmetic programs yields both succinct communication complexity, i.e. the response to the common challenge consists of only 3 encrypted elements (6 Paillier ciphertexts), and efficient response computation and key derivation, i.e. only linear to the QAP size.

Our scheme is particularly suitable for private auctions in financial intermediation in which one party wants to privately communicate with another party about committed financial information which satisfies a relation R of interest. It is also usable in other applications such as biometric-data sharing.

Our new notions, i.e. Witness-Key-Agreement and Split Designated Verifier NILP may be of independent research interest as well as interesting application of NILP.

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